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- PRINCIPLES AND APPLICATIONS -

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THE ULTRASONIC TESTING OF PAPER AND BOARD - PRINCIPLES AND APPLICATIONS -

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ABSTRACT

The basic ideas involved in the ultrasonic testing of paper and board have been reviewed. Advances in transducer and instrument design, as well as instrumentation, have enabled a once time consuming laboratory measurement to be fully automated. Significant advances have also been made in developing on-line ultrasonic testing for the papermachine.

There has been considerable growth in the number of applications for ultrasonic testing which we have briefly reviewed. These areas include measurement of in-plane and out-of-plane elastic properties, and their correlation with failure properties, i.e., in-plane tensile strength, compressive strength, and ZD strength. The use of polar diagrams as a diagnostic tool for the paper machine is another important development as it relates to headbox design and the problems of paper curl and combined board warp.

Other applications included: the non-destructive measurement of the aging of paper, and the characterization of small samples of wood and paper. Ultrasonic testing also shows promise for evaluating tissue softness.

Improvements in ultrasonic testing and reliability are expected to continue, particularly for Z-direction measurements, and for determining more information about the viscoelastic behavior of paper and the presence of defects.

KEYWORDS
ultrasonics, elastic properties, modulus, strength properties, aging, softness, mechanical testing.

INTRODUCTION

The benefits of non-destructive testing are obvious in many areas of material science, and paper is no exception. The particular non-destructive measurement we will focus on is ultrasonic testing. We are concerned with the measurement of the speed of a sonic wave in paper and the factors controlling it.

However, what is of real interest is the fact that the wave speed in a material is directly related to its elastic properties. To a good first approximation the speed of a longitudinal wave is given by:

$$v = \sqrt{C/\rho} \qquad (1)$$

where v is the speed of sound, and C/ρ is the specific elastic constant in the direction in which the wave is travelling.

Sonic waves can be one of two types namely longitudinal or shear waves. This allows us to measure both the longitudinal and shear elastic constants, and from these measurements we can calculate the engineering moduli as we shall see shortly.

Paper is a highly directional material (anisotropic) meaning that its properties vary with direction. We are familiar with the fact that MD (machine direction) and CD (cross machine direction) properties are usually different, and also that properties in the thickness or ZD direction (difficult to measure mechanically on a routine basis) are very different from those in the MD and CD.

In materials where the elastic properties are independent of direction (isotropic) only two elastic constants are required to characterize the material. A large number of elastic constants, in the most general case ninety one, are required to characterize an anisotropic material, but since paper can be approximately treated as an orthotropic plate material, i.e., one having three planes of symmetry, then the number of elastic constants can be reduced to nine. Ultrasonic measurements have recently made it possible to determine all nine elastic constants (1).

Another important point regarding paper is that it exhibits time dependent (viscoelastic) behavior. A familiar consequence of this is that paper creeps or stretches under a constant load. However, the consequences of this time dependent behavior for ultrasonic measurements are more subtle.

The time dependent behavior of paper, even at very small strains typical of ultrasonic testing, is basically controlled by relaxation processes taking place at the molecular level. These can occur in cellulose, hemi-cellulose, and lignin, the main chemical components of the fiber. Temperature, moisture, and the time scale of testing, also influence this time dependent behavior. The time scale of testing is particularly important when we try to understand the relationship between mechanical (relatively long times) and ultrasonic testing (relatively short times) which we will discuss shortly.

In-plane Ultrasonic Measurements

In principle, measuring the speed of sonic wave propagation is straightforward, i.e., the time it takes for the wave to travel a known distance. In early testing the time it took for the wave to travel between the transmitter and receiver for a number of transducer spacings was determined. A plot of the data resulted in a straight line which did not pass through the origin (2). This has been attributed to delays in triggering and transducer coupling with the paper, otherwise a single fixed position measurement would have sufficed. Sonic wave velocity was determined from the slope of this graph.

Typically, in-plane velocity measurements at IPST are determined using two transmitters and one receiver, and a cross correlation technique is used to automatically calculate the wave travel time (3). Other means for measuring sonic velocity include a rod resonance technique (4), and a non-contact plate wave resonance technique (1). In the three transducer technique shear waves are created by simply rotating the transmitters through 90°.

The frequency of the in-plane transmitted wave is typically 60 kHz and the wave length is long so that under these circumstances paper can be treated as a continuum, i.e., the individual fibers and bonds are not effectively seen. However, because of the viscoelastic nature of paper the transmitted wave is distorted, and this distortion can be analyzed to determine the viscoelastic behavior of paper (4)(5).

Out-of-Plane Ultrasonic Measurements

A major development at IPST has been the establishment of techniques for measuring sonic wave velocities in the thickness direction of paper (5)(6). Two basic measurements are required for the calculation of the out-of-plane longitudinal elastic constant: C_{33}/ρ or modulus E_z/ρ. These are an effective caliper, and the wave travel time for the propagation of longitudinal waves. In the case of longitudinal waves a soft platen caliper technique not only gives a more meaningful measurement of caliper, but is a very effective way of coupling ultrasound energy into paper at the frequencies employed 1 MHz. This technique has been automated, and user friendly software makes it very easy to use.

Out-of-plane shear measurements are reasonably straightforward, and again the two basic measurements required are caliper and wave travel time. For smooth samples, i.e., roughness $r < 0.04$ (r = (hp caliper-sp caliper)/sp caliper), a hard platen technique can be used. However, for rough samples a more time consuming pillow technique has to be employed (5).

Although procedures for out-of-plane sonic wave velocity measurements are well developed, and include the more recent wheel (6) and fluid filled wheel technique (7), there dependence on papermaking and converting variables still requires further understanding.

Mechanical versus Ultrasonic Testing

We have already mentioned that there are differences between mechanical and ultrasonic testing. When a tensile strip of paper is tested at a constant strain rate in a mechanical tester it undergoes a lateral contraction, and the ratio of this lateral contraction to the extension of the sample, in the direction of straining, is Poisson's ratio. However, when a longitudinal sonic wave is propagated through paper there is no contraction or Poisson's ratio effect. In mechanical testing we determine an engineering or Young's modulus E (or specific modulus E/ρ), and in ultrasonic testing, as given by equation 1 above, we determine a specific elastic constant, and the difference between them is simply the Poisson's ratio effect. It can be shown that the engineering modulus for an isotropic material is calculated as follows:

$$E/\rho = (C/\rho)/(1 - \nu^2) \qquad (2)$$

where ν is the Poisson's ratio, typically around one third. Therefore, the engineering modulus E is lower than the elastic constant C by about 10%.

A convenient ultrasonic wave propagation method for determining Poisson's ratios has been derived by Baum and Bornhoeft (8), and their simplified equation for Poisson's ratio with respect to MD CD, i.e, ν_{xy}, is given below.

$$\nu_{xy} = -1/A + 1/A\ (1 + A^2/R - A/R) - A)^{1/2} \qquad (3)$$

where $A = (v_x/v_{45})^2$, $R = (v_x/v_y)^2$, and $\nu_{yx} = R\ \nu_{xy}$. v_x and v_{45} are the longitudinal velocity in the MD and shear velocity at 45° respectively.

When we compare E/ρ measured mechanically with that measured ultrasonically we find that the ultrasonic engineering modulus is 25% to 40% higher than the mechanical modulus. The main reason for the difference is the very different time scale that is involved in mechanical and ultrasonic testing, and the fact that paper is a viscoelastic material. Explanations of this have been provided by Habeger (9) and Batten (10). At very short times the polymer chain does not have time to adjust its configuration in response to the applied strain, and consequently the resistance to deformation is much greater than it is at much longer times when it is almost completely relaxed.

In the case of mechanical testing the paper is strained at a constant rate, i.e., a ramp function, while in ultrasonic testing a sinusoidal strain is applied, and these differences further complicate the comparison. Let us now estimate the time scale of testing in both cases.

Mechanical Testing: TAPPI test method uses a 15 mm wide strip of paper with a 100 mm separation between the jaws. The rate of jaw separation or strain rate is 25 mm/min. Under these conditions a paper having a failure strain of say 3% would take 7.2 seconds to fail. However, if we are interested in very small strains, i.e., less than 0.1%, then the time is 0.24 seconds.

Ultrasonic Testing: Now an in-plane ultrasonic wave travels through paper with a velocity v which is equal to the product of the wave frequency and wavelength. The period of the wave T is equal to the inverse of the frequency. Therefore, for a wave having a frequency of 60 kHz and wave speed of between 1 km/sec and 5 km/sec (approximate range for paper), the wave length is between 1.67 cm and 8.3 cm. The time scale or period is 1.67×10^{-5} seconds.

We see that the time difference between mechanical and ultrasonic testing is several orders of magnitude. Therefore, with mechanical testing the relaxation processes which occur at much shorter times do not contribute. Any attempts to model this time dependent behavior with springs and dashpots must include the whole spectrum of relaxation times. Therefore, relaxation times derived from mechanical testing alone will not suffice.

Moisture and Temperature Effects

We have already noted that paper is a viscoelastic material, and as a consequence the elastic properties are both temperature and moisture dependent. The effects of moisture and temperature on sonic wave propagation have been studied by Berger, Habeger, and Pankonin (11).

A linear equation correcting for moisture and temperature over the range of 2 -18% m.c. and 20 - 80 T°C was found for a number of papers and cellophane (11).

$$V^2 = \beta_m (M - M_o) + \beta_T(T - T_o) + V_o^2$$

For the case of linerboard (11) the rate of loss of elastic constant is greater in the CD than the MD, and therefore the elastic in-plane anisotropy ratio, $R = E_{md}/E_{cd}$, will increase with increasing moisture content. Using the above equation the change in sheet anisotropy with increasing moisture content can be predicted by the following equation:

$$R(=E_{md}/E_{cd})_{moisture} = R_o\{(\beta_m/V_o^2)_{MD}(M - M_o) + 1\}/\{(\beta_m/V_o^2)_{CD}(M - M_o) + 1\} \quad (4)$$

where V, M, T, β_m, and β_T, are, respectively, the velocity, moisture content, temperature, and moisture and temperature regression coefficients (11). Suffix o denotes the chosen reference conditions, i.e., a moisture content at 50% RH and a temperature of 25°. conditions.

Waterhouse and Charles (12), and Salmen and Rigdahl (13) found that refining and wet pressing, normally bond making processes, reduce elastic anisotropy while supercalendering increases elastic anisotropy (12). Increasing moisture content was also found by Schulgasser and Page (14) to increase elastic anisotropy.

APPLICATIONS

* Papermaking process:

Non-destructive testing has the potential for on-line process monitoring, the value of which includes a more uniform product, improved product quality, and significant economic benefits (15)-(17).

A change in the elastic properties at the dry end of the papermachine may be attributed to one or more process changes such as refining, additive addition, forming, wet pressing, drying, and calendering. These changes may impact the elastic properties of the fiber, interfiber bonding, and sheet structure variables controlling the elastic properties of the sheet. These relationships have been reviewed by Habeger and Baum (15) and Baum (18).

The use of ultrasonics to monitor certain aspects of the forming process is currently receiving a lot of attention. As we have already stated paper is anisotropic. The in-plane anisotropy is mainly controlled by fiber orientation and drying restraints. In addition to MD and CD property ratios more complete information is given in the form of polar plots, which are readily derived using ultrasonic techniques (3)(19). In many instances it is found that the principle elastic axes do not coincide with the MD and CD, but are rotated through a certain angle known as the lean or polar angle. Factors which may be responsible for this deviation include flow imbalances and cross flows from the headbox.

Under conditions of complete MD and CD restraint during drying, the effective fiber modulus will be approximately constant, and under these circumstances the shape of the polar diagram will be mainly controlled by fiber orientation. However, it should be emphasized that the shape of the polar diagram is not necessarily the shape of the fiber orientation distribution function, i.e., a polar plot of fiber orientation. To illustrate this point we have assumed that the in-plane shear modulus is invariant with direction and this enables us to calculate both the fiber orientation and elastic constant variation with polar angle. In this case the elastic anisotropy ratio $R(=E_{md}/E_{cd})$ completely characterizes the sheet as shown by Schulgasser (20). Figures 1 and 2 show polar plots of fiber orientation and elastic constant for R = 3. Fiber orientation distributions are not easy to measure and generally dyed fiber techniques only relate to the surface layers of the sheet.

(place Figures 1 and 2 near here)

Generally, fiber orientation and modulus distributions change continuously from the wire side to the felt side of the sheet. Although there may not be a resultant polar or lean angle for the whole sheet there can be significant differences in polar angle between the wire and felt halves of the sheet (21).

* Failure properties: Tensile & Compressive strength

Tensile Strength

Ultrasonic techniques have sometimes been referred to as strength sensors. This is clearly a misnomer since only elastic properties are being measured. Generally, as the elastic properties of a material increase so do their strength properties. However, caution should be exercised in expecting a one to one relationship between failure and elastic properties for paper. The ultimate strength of paper depends on a number of variables, and various models have been proposed for predicting it (22),(23).

In considering in-plane tensile strength one relationship between strength and modulus is Griffith's equation derived for the fracture of a brittle material. Its application to paper has been studied by several researchers. Waterhouse and Bither (24) found the following correlation:

$$(\sigma/\rho)_{tensile} = (E/\rho \times G/\rho)^n \quad (5)$$

where σ/ρ, E/ρ, and G/ρ are the specific tensile strength, modulus, and fracture toughness, respectively. In practice it is possible, that over a limited range of machine operating conditions for a given product, that G/ρ is constant.

A good correlation has been found by Baum et al (16) between z-direction tensile strength and modulus. Waterhouse (25), and Waterhouse and Bither (24) have also found this relationship to be independent of whether bonding is achieved by refining or wet pressing. Nevertheless, more research is required to better understand the factors which control the z-direction strength of paper.

Compressive Strength

Compressive strength is one important failure property where we would expect a strong correlation between it and elastic properties, since buckling is the main mode of failure. Habeger and Whitsitt (26) have developed a basic model and equations for predicting compressive strength in terms of the in-plane and out-of-plane elastic constants.

Their full equation for MD compressive strength is given as follows:

$$(\sigma/\rho)_{comp} = ((C_{11}/\rho)^{2/3} \times (C_{55}/\rho)^{1/6} \times (C_{33}/\rho)^{1/6})/RW \quad (6)$$

where C_{11}/ρ, C_{55}/ρ, C_{33}/ρ are the MD in-plane longitudinal, out-of-plane shear, and out-of-plane longitudinal elastic constants respectively. The roughness/weakness factor is given by $RW = (\delta A_o/t) \times (C_{55}/\tau_f)$ where $(\delta A_o/t)$ is initial deflection to thickness ratio of critical lamina, and τ_f is the out-of-plane shear strength.

This equation can be simplified by assuming that the RW factor remains constant, and that:

$$C_{55}/\rho = \text{constant} \times ((C_{11}/\rho \times (C_{33}/\rho))^{1/2}.$$

thus equation (6) becomes:

$$(\sigma/\rho)_{comp.} = \text{constant} \times (C_{11}/\rho)^{3/4} \times (C_{33}/\rho)^{1/4} \quad (7)$$

similarly for CD we have that

$$(\sigma/\rho)_{comp.} = \text{constant} \times (C_{22}/\rho)^{3/4} \times (C_{33}/\rho)^{1/4} \quad (8)$$

The ratio of MD/CD compressive strength then becomes:

$$R_c (= \sigma_{md}/\sigma_{cd})_{comp.} = (C_{11}/\rho)/C_{22}/\rho)^{3/4} \qquad (9)$$

A comparison of this prediction with experiment is shown in Figure 3, and we see that the agreement is quite good.

(place Figure 3 near here)

For handsheets having an in-plane random fiber orientation $C_{11} = C_{22}$, equation (7) or (8) can be written in terms of engineering moduli as follows.

$$(\sigma/\rho)_{comp.} = \text{constant} \times (E/\rho)^{3/4} \times (E_z/\rho)^{1/4} \qquad (10)$$

The variation of compressive strength for handsheets made from different pulps and additives using equation (8) is shown in Figure 4.

(place Figure 4 near here)

Although the correlations are quite good (see summary table of regression analysis) we note that a universal correlation is not evident. It is possible that the above simplifications are not valid, and that the RW factor is not constant. For example, low levels of strength additive have little effect on modulus, although the shear strength is expected to increase significantly, and therefore the roughness/weakness factor will not be constant.

* Aging and Endurance Characteristics of Paper

Monitoring the aging of paper non-destructively has been the subject of a review by Waterhouse (27). It is considered desirable to have a means of determining non-destructively the extent to which valuable documents and art artifacts have suffered strength degradation either due to poor papermaking, and/or poor storage conditions. In some instances degradation gives rise to embrittlement, resulting in an increase in initial modulus, but a concomitant loss in strength and elongation (see for example Figure 3 (27)). In such circumstances, if only the initial modulus is measured then the increase might be misinterpreted as a stronger paper. One possible route to avoiding the latter situation would be to measure changes in the viscoelastic behavior of the paper (27).

In another investigation Waterhouse and Barrett (28) used non-destructive testing to determine differences in in-plane and out-of-plane elastic properties for European papers in good and poor condition. The papers had been sized with gelatin, which produced a significant improvement in elastic properties. As a result of acid hydrolysis and/or poor storage conditions a very distinct difference in the elastic properties of the papers in good and poor condition was found, and was attributed to a deterioration in gelatin reinforcement.

* Wood Coupons and Mini-Handsheets

Ultrasonic wave propagation techniques can also be useful in characterizing relatively small samples of wood and paper. We have used out-of-plane longitudinal and shear apparatus developed at IPST to measure the in-plane and out-of-plane elastic constants of wood coupons (16 mm x 16 mm x 1 mm) cut from a 36-year-old white spruce log, and mini-handsheets (19 mm diameter) made from the individual pulped wood coupons. The main objective of this investigation was to better understand the compressive strength potential of wood pulp fibers. The relationship between the elastic properties of the wood and their subsequent modification by pulping, to the short span compressive strength of mini-handsheets made from the pulped wood coupons, was investigated on the basis of the Habeger-Whitsitt compressive strength model already discussed above.

Using the equipment shown in Figure 5, we have found that lignin removal, using acid chlorite delignification, increases the in-plane elastic constants C_{11} and C_{22}, and decreases the out-of-plane elastic constant C_{33}. Figure 6 shows the variation of the in-plane elastic constant with apparent density for mini-handsheets made from the pulped wood coupons. Also shown is the variation predicted using the measurements made on the wood coupons. This underpredicts the measured variation and is attributed to the lower density and lack of fiber connectedness in the pulped wood coupons.

(place Figures 5 and 6 near here)

The variation of specific compressive strength with mini-handsheet apparent density is shown in Figure 7. The mean specific compressive strength is 37.1 Nm/g

(place Figure 7 near here)

* Tissue softness

Softness, as typical of many paper properties, is controlled by a number of factors, and Pan, Habeger, and Biasca (29) have demonstrated that longitudinal out-of-plane ultrasonic wave propagation techniques may be useful in assessing tissue softness. In fact it appears that high frequency non-destructive measurements offer a significant advantage over mechanical testing.

Two categories of softness were defined: a) bulk softness associated with crumpling a tissue and b) surface softness associated with the feel or tactile response of the tissue.

Using pair comparison tests to quantitize softness, it was found using multiple regression analysis, that it strongly correlated with ultrasonic impedance Z, mass specific attenuation A/W, and grammage W.

$$\text{SOFTNESS} = aZ + bA/W + cW + d \tag{11}$$

The regression constants a,b,c, and d for bulk and surface softness are given in (29) for which $R^2 \geq 0.785$.

The ultrasonic impedance is simply the grammage divided by the time of flight, and does not involve a caliper measurement. The attenuation coefficient is obtained from a Fourier analysis of the transmitted and received (after the wave has passed through the tissue) waveforms. The ultrasonic measurements in this application are made at a pressure of 20 kPa rather than 50 kPa.

CONCLUDING REMARKS

The basic ideas involved in the ultrasonic testing of paper and board have been presented. The pioneering work of Craver and Taylor (2) in establishing these procedures is now well recognized. Advances in transducer and instrument design, as well as instrumentation, has enabled a once time consuming laboratory measurement to be fully automated. Significant advances have also been made in developing on-line ultrasonic testing for the papermachine. It is also possible that certain converting operations may also benefit from similar instrumentation.

There has also been considerable growth in the number of applications for ultrasonic testing which we have briefly reviewed. These areas include measurement of in-plane and out-of-plane elastic properties and their correlation with failure properties, i.e., in-plane tensile strength, compressive strength, and ZD strength. The generation of polar diagrams as a diagnostic tool for the paper machine is another important development as it relates to headbox design and the problems of paper curl and combined board warp.

Other applications included the non-destructive measurement of the aging of paper, and the characterization of small samples of wood and paper. Ultrasonic testing also shows great promise for evaluating tissue softness.

Improvements in ultrasonic testing and reliability are expected to continue, particularly for Z-direction measurements, and for determining more information about the viscoelastic behavior of paper and the presence of defects.

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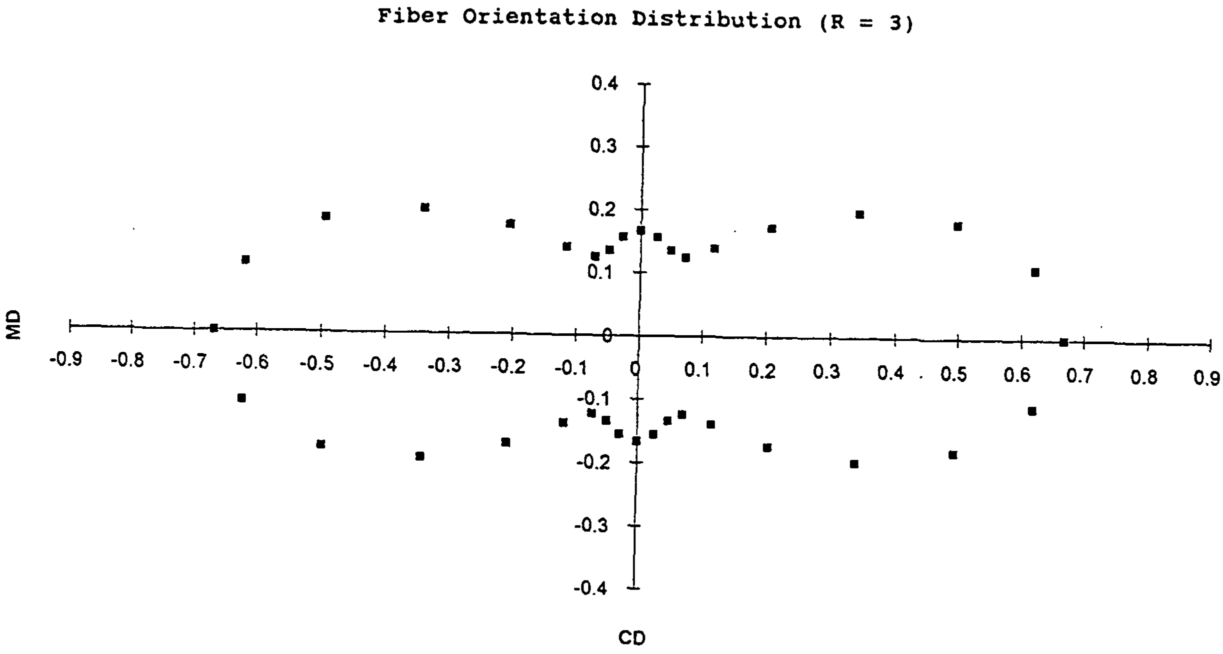


Fig 1. Polar Diagram of Fiber Orientation Distribution Function Assuming a Constant In-plane Shear Modulus.

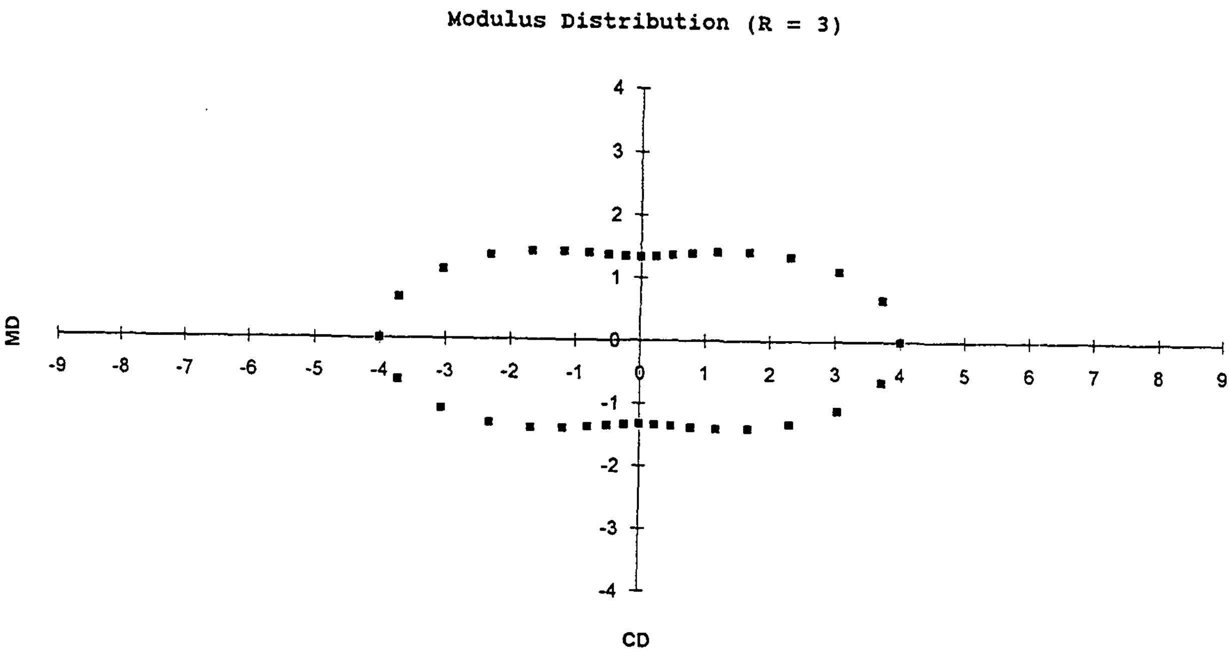
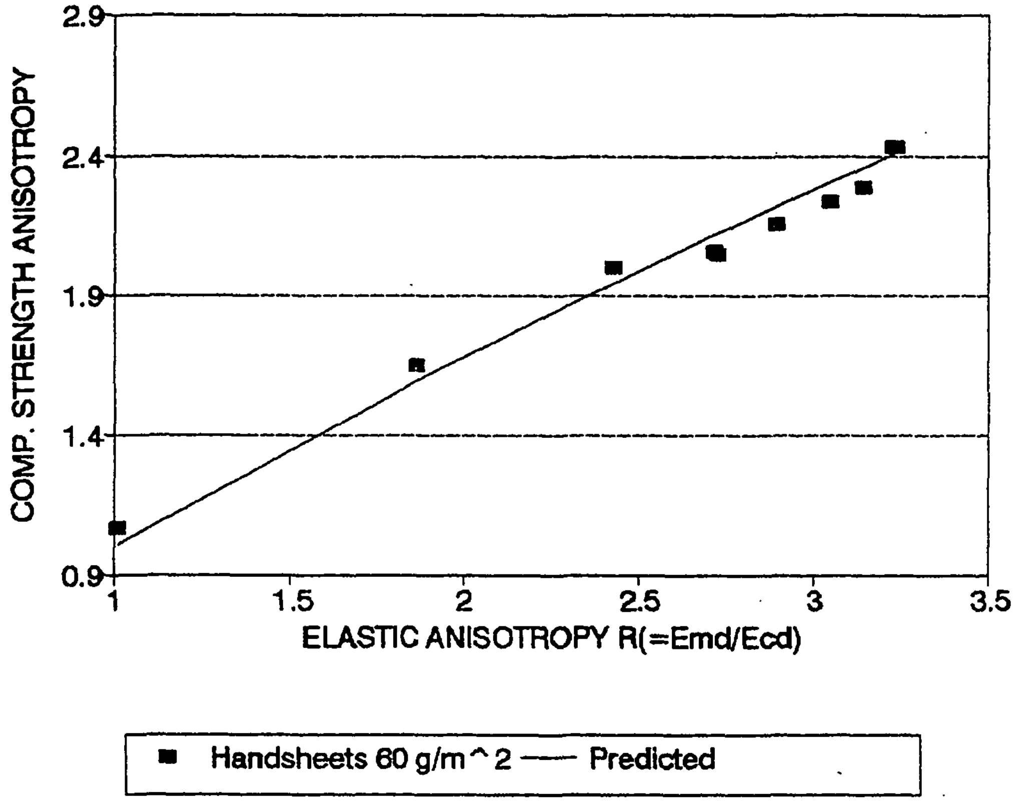


Fig 2. Polar Diagram of Elastic Modulus Assuming a Constant In-plane Shear Modulus.



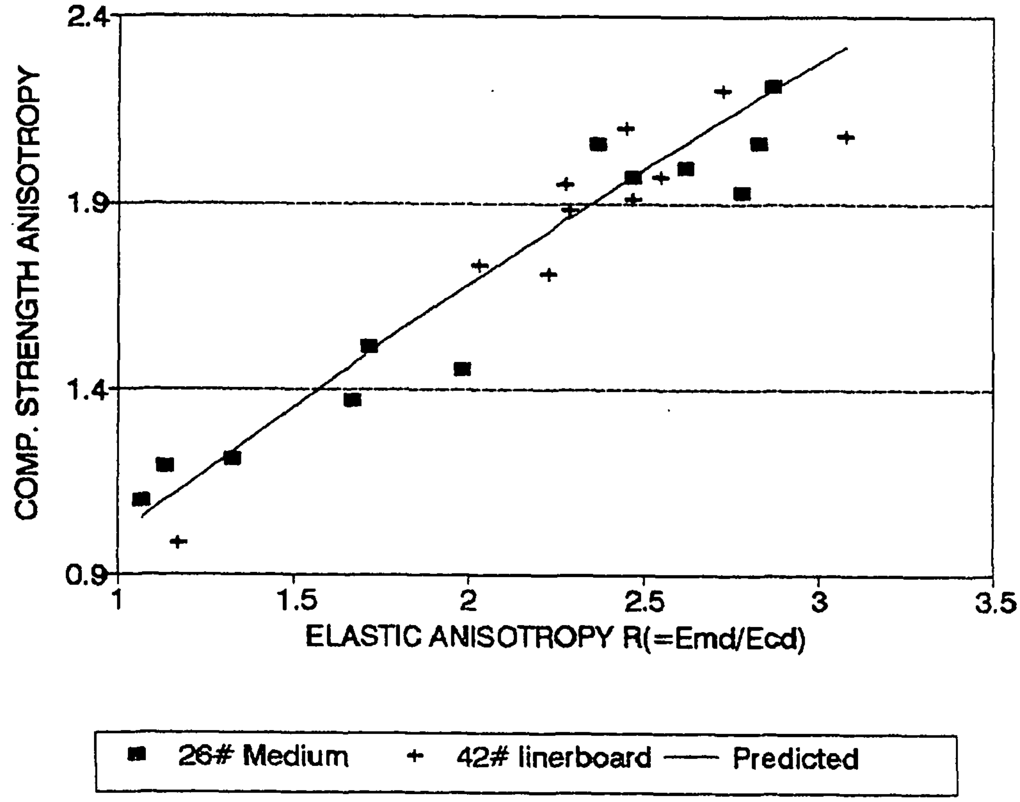
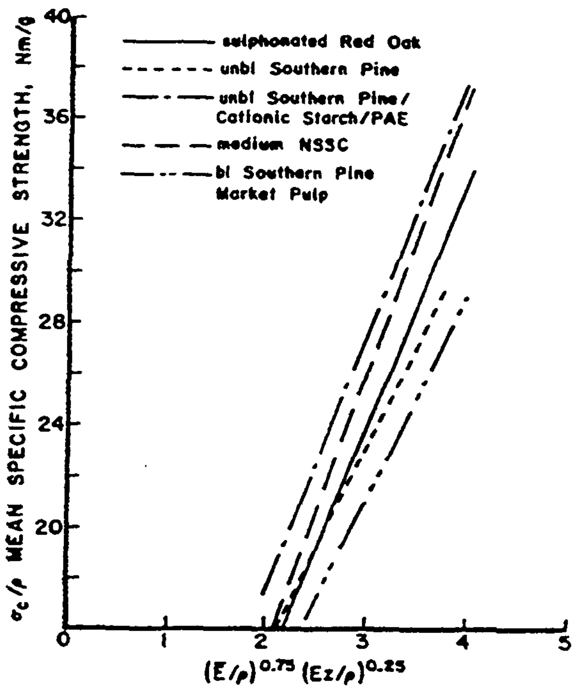


Fig 3. Variation of Compressive Strength Anisotropy with Elastic Anisotropy.



Summary of Regression Equations.

Pulp Type	Slope	Intercept	r^2	
Unbl. Southern Pine	7.485	0.1667	0.974	(65% yield)
Bl. Southern Pine	7.982	-2.984	0.975	—
Sulphonated Red Oak	9.507	-4.612	0.961	(85% yield)
NSSC Medium	10.81	-6.437	0.992	(commercial medium)
Unbl. Southern Pine/ Cationic Starch/PAE	10.0	-2.62	0.947	(65% yield)

Fig 4. Correlation of Specific Compressive Strength with Elastic Properties For Different Furnishes.

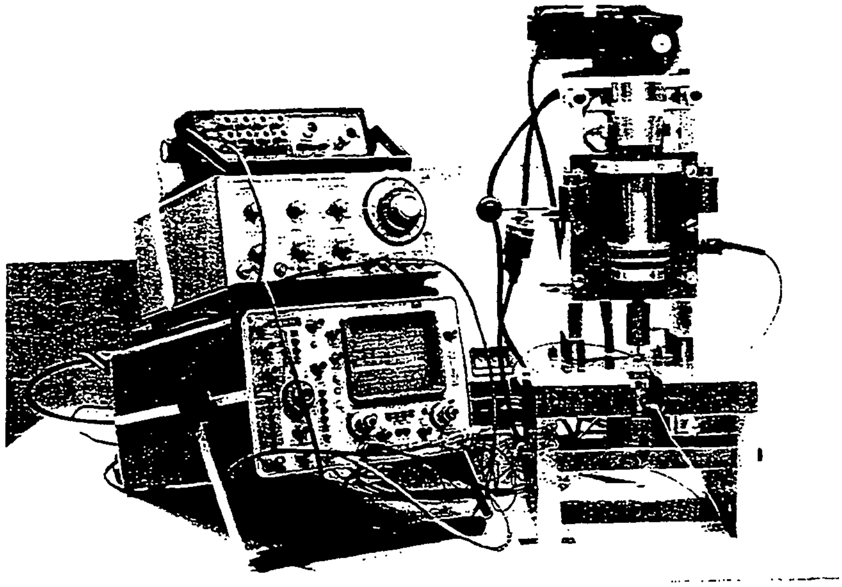


Fig 5. Out-of-plane ultrasonic equipment for measuring the In-Plane and Out-of-Plane Elastic Properties of small Wood Coupons and Mini-handsheets.

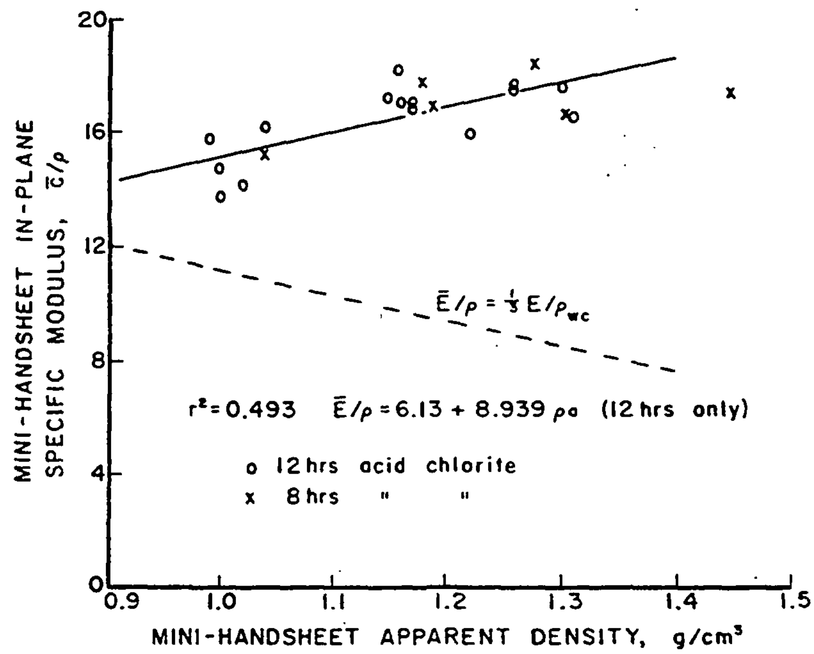


Fig 6. Variation of In-plane Elastic Constants with Apparent Density for Mini-handsheets.

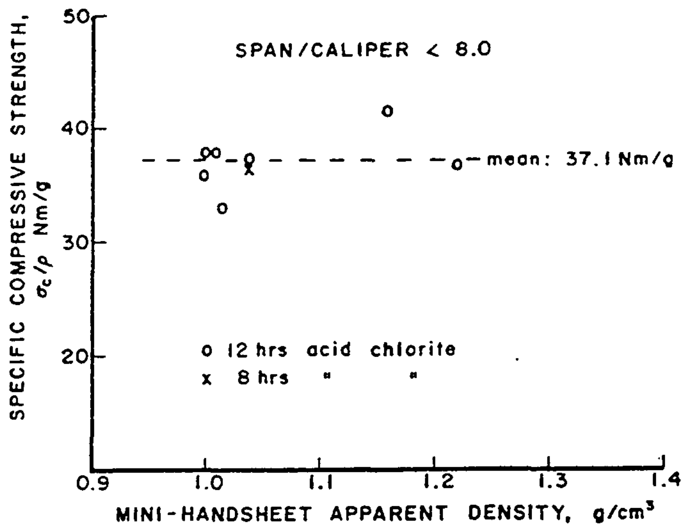


Fig 7. Variation of Specific Compressive Strength with Apparent Density for Mini-handsheets.